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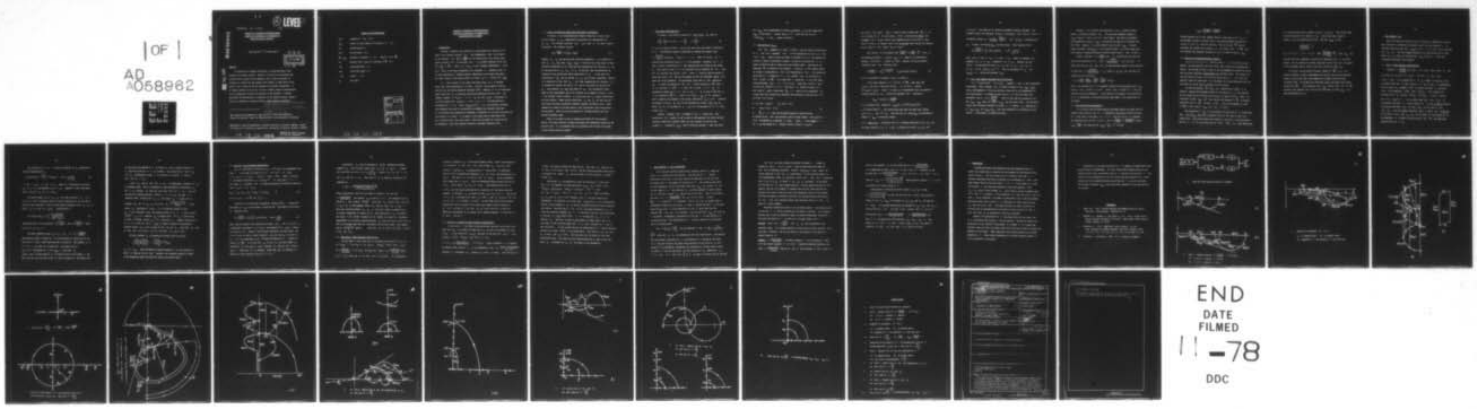

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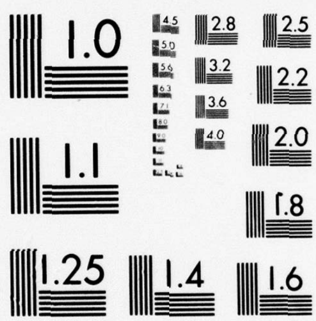

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BLENDING OF UNCERTAIN NONMINIMUM-PHASE PLANTS FOR ELIMINATION OR REDUCTION OF NONMINIMUM-PHASE PROPERTY

Isaac Horowitz †* and Amos Gera *

Abstract

The benefits of feedback achievable in nonminimum-phase (nmp) systems are severaly limited. However, if there are two parallel nmp branches whose outputs can be measured, then it may be possible to eliminate or reduce the nmp property with respect to some parts of the system, even if each of the two branches has uncertain parameters. It is always possible to do so for a finite, nonzero range of uncertainties for any complexity of pole-zero patterns of the branches. A design philosophy and methodology is presented for the general problem, permitting one to approach an optimum solution, under various constraints.

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SYMBOLS AND ABBREVIATIONS

$B(\omega)$	boundary of $T(\omega)$ (2.1)
$\mathcal{E}_\psi$	*number of encirclements of origin by* ψ (2.)
mp	minimum-phase (1.)
mps	mp and stable (2.1)
$\mathcal{N}$, $F_{\mathcal{N}}$	boundary of quadrant 1 (2.), map of F on $\mathcal{N}$.
NL	Nyquist locus — map of a function on $\mathcal{N}$ (2.)
nmp	nonminimum-phase (1.)
rhp	right half plane (1.)
$T(\omega)$	$\{P(j\omega)\}$ (2.1)
θ_P	Arg $P(j\omega)$.

BLENDING OF UNCERTAIN NONMINIMUM-PHASE PLANTS FOR ELIMINATION OR REDUCTION OF NONMINIMUM-PHASE PROPERTY

1. INTRODUCTION

Consider a feedback loop containing a constrained part (denoted as the Plant) with transfer function $P(s)$, and compensation $G(s)$ to be chosen by the designer. Let $S_P^T(s) \triangleq \frac{\partial T/T}{\partial P/P}$ be the sensitivity of any system input-output function $T(s)$ to the plant $P(s)$. It is known (Horowitz 1963) that $S_P^T = \frac{1}{1+L_P}\left(1 - \frac{T_o}{T}\right)$, where $T_o = (T)_{P=0}$ is the 'leakage transmission'. Hence $L_P(s)$ which is the "loop transmission for reference P" (Horowitz 1963) is of great importance in feedback design, especially in the large class where the plant output is the system output, for which $T_o = 0$, giving $S_P^T = \frac{1}{1+L_P}$.

If P has no zeros in the right half-plane (rhp), i.e., is minimum-phase (mp), then theoretically, at least, $|L_P(j\omega)|$ can be made as large as desired over any finite bandwidth (Horowitz 1975). However, if P is nonminimum-phase (nmp) then L_P is severely constrained (Horowitz and Sidi 1978). For example, suppose $P = P_m(s-a)$, $a > 0$, P_m mp and $L_P(j\omega)$ is designed in the usual manner with $|L_P(j\omega)| > 1$ for $\omega < \omega_c$, < 1 and monotonically decreasing for $\omega > \omega_c$. Then $(\omega_c)_{max} \approx 0.5a$, i.e., the available feedback loop bandwidth and with it the benefits of feedback are severely curtailed. It is therefore very important to eliminate, if possible, or at least alleviate the nmp property in a plant. It is shown in this paper that to some extent and for a certain class, this can be done. Note that rhp poles of a plant do not, by themselves, limit the feedback bandwidth achievable (Horowitz 1963).

1.1 A Class of Plants for which nmp Alleviation is possible.

In Figure 1, the constrained part (darker lines) has a single input x and two outputs y_1, y_2 independently measurable and compensatable by G_a, G_b. The transfer functions Y_1/X, Y_2/X have P_1 in common (usually the poles). The transfer function

$$P_{eq} = \frac{Z(s)}{X(s)} = P_1(P_aG_a + P_bG_b) \qquad (1)$$

Suppose P_a, P_b are nmp and with uncertain parameters. Is it possible to find G_a, G_b such that $(P_aG_a + P_bG_b)$ is mp over the entire range of these uncertain parameters? If so, then the loop transmission for reference P_1 has no rhp zeros due to P_a, P_b. If P_1 is also mp, then the system mav be designed to have arbitrarily small sensitivity to P_1. In any case, the limitations on $S^T_{P_1}$ due to the nmp P_a, P_b are at least eliminated. However, it is important to note that L_a, the 'loop transmission for reference P_a', is obtained by cutting the loop at aa' and so does have the rhp zeros of P_a, and similarly L_b must have those of P_b. The incremental linear model of the longitudinal axis of an aircraft is an example of Figure 1, with x the elevator control surface, y_1 pitch rate and y_2 normal acceleration at pilot station. Under certain conditions, P_a and P_b are both nmp and with highly uncertain parameters (Edwards, Rediess and Taylor 1970). Other examples are common in process control, with time delays often constituting the nmp paths. These may be well modelled by all-pass networks over any desired frequency range.

There is no single or set of formulae available for this problem. Rather, this paper presents a design philosophy and methodology usable for any specific case. It is developed here by proceeding step by step from simple to more complex problem classes.

2. THE SINGLE POLE-ZERO PAIR

In Figure 1, the zeros of concern of $(P_aG_a + P_bG_b)$ are those of

$$\left\{ \frac{G_a}{G_b} + \frac{P_b}{P_a} \triangleq F + P \triangleq \psi \ , \quad F = \frac{G_a}{G_b} \ , \quad P = \frac{P_b}{P_a} \right\} \tag{2}$$

F is to be chosen so that ψ has no rhp zeros over the range of uncertainty of P. Considerable insight is obtained by studying the simple class

$P = \dfrac{K_1 k(s-z)}{(s-\lambda z)} \triangleq K_1 k P_1(s)$, $K_1 k, z > 0$, $\lambda > 1$, with $k \in [1, k_2]$, $k_2 > 1$

denoting the uncertainty only in k in the meantime. Hereafter $K_1 = 1$ is used, so that in practice the F that emerges must be divided by the actual K_1. Note from (2) that if F is a solution for an uncertain set $\{P\}$, then $1/F$ is a solution for the set $\{1/P\}$. Also, F need not have an excess of poles over zeros. Let $\mathcal{N}$ be the boundary of half the forbidden region — the upper rhp (first quadrant). By the Nyquist locus (NL) of $H(s)$ denoted by $H_{\mathcal{N}}$, is meant the mapping of $H(s)$ as s moves clockwise on $\mathcal{N}$ from 0 to jR and then $s = Re^{j\theta}$, θ from $\pi/2$ to zero, $R \to \infty$. $P_{\mathcal{N}}$ is sketched in Figure 2a at $k = 1$ and $k = k_2 < \lambda$. It is easy to see that if $-F$ is a constant $\in (\frac{k_2}{\lambda}, 1)$, then $\forall\, k \in [1, k_2]$, $\psi_{\mathcal{N}}$ of (2) executes half a negative encirclement of the origin (denoted by $\mathcal{E}_\psi = -0.5$). Invoking the Nyquist criteria: N_ψ = No. of encirclements of origin $= 2\mathcal{E}_\psi = N_z - N_P$ where N_z = no. of rhp zeros of ψ, N_P = no. of rhp poles of $\psi = 1$ here, giving $N_z = 0$.

However, a dynamic $F(s)$ is needed if $k_2 > \lambda$ (Figure 2b). The restriction $\ell n\, F$ regular in rhp is made in the meantime, for which the Bode integrals (Bode 1945, Horowitz 1963) are very helpful in deriving sup optimum F, denoted by F_{sup} — since a strictly optimum F does not exist

but F_{sup} can be approached as closely as desired. It is next shown that $F_{sup} = (s+\lambda z)/(s+z)$, giving $\sup k_2 = \lambda^2$, with the root loci of $1+kP_1/F_{sup}$, $P = kP_1$, shown in Figure 4.

2.1 Derivation of F_{sup}.

Let $T(\omega)$ (template of $P(j\omega)) = \{P(j\omega)\}$ over the range of uncertainty of P, and $B(\omega)$ = Boundary $T(\omega)$. If the uncertainty is only in k then $T(\omega) \equiv B(\omega)$, some of which are shown in Figures 2b, 3a, b the numbers and letters in order of increasing ω values. The NL of a satisfactory $-F$ is shown in Figure 2b and in Figure 3b on the Nichols chart. (Henceforth the subscript $\mathcal{N}$ is omitted in the figures.) $F(s)+P(s)$ is the complex number (or vector), originating at $-F(s)$ and terminating at $P(s)$. By considering the motion of P relative to F, i.e., taking $-F(s)$ as the origin of $(F+P)$ at each $s \in \mathcal{N}$, it is seen that P encircles $-F$ one half times counterclockwise, $\mathcal{E}_\psi = -.5$, $\forall k \in [1,k_2]$. Note that $\text{Arg}(-F) > \text{Arg } P$ in Figure 2b, except at $\omega = 0, \infty$ and that a $-F$ similar to above, but with $\text{Arg}(-F) < \text{Arg } P$ would give $\mathcal{E}_\psi = .5$, which is not acceptable. It is seen that the following are necessary and sufficient conditions for $F+P$ to be mp $\forall k \in [1,k_2]$.

A. (a) $-F(0) > k_2P(0)$. (b) $-F(\infty) < P(\infty)$

B. $-F(s) \neq T(s) \quad \forall s \in$ (3)

C. $\mathcal{E}_\psi = -.5$, with the following geometric interpretation:

In Figure 3b the $B(\omega)$ are vertical lines of equal length. Each point on $B(\omega)$ corresponds to a specific k value, $B_k(\omega)$. At any fixed k, let n be the number of ω values at which $|F(j\omega)| = |B_k(\omega)|$,

Arg $-F(j\omega) <$ Arg $B_k(\omega)$. Then n must be even in order that $\mathcal{E}_\psi = -.5$, as in Figures 3a, b. In other words, $-F(j\omega)$ in moving from above $B(0)$ to below $B(\infty)$, as per (3Aa,b), may temporarily be on the left of $B(\omega)$ (Arg $-F <$ Arg P) in Figures (3a,b), but in the net, must execute the passage on the right of $B(\omega)$ where Arg $-F >$ Arg P .

From (3Aa,b), it is necessary that $\left|\frac{F(0)}{F(\infty)}\right| > k_2 \frac{P(0)}{P(\infty)} = \frac{k_2}{\lambda}$, with k_2 the maximum tolerable k (recall $k_{min} = 1$). Hence it is desirable to maximize $|F(0)/F(\infty)|$, subject to (3B,C). However, for mp and stable (denoted by mps) $F(s)$ (Horowitz 1963)

$$\ell n \left|\frac{F(0)}{F(\infty)}\right| = -\frac{\pi}{2}\int_0^\infty \frac{\theta_{-F}(\omega)d\omega}{\omega} \ , \quad \theta_{-F}(\omega) \triangleq \text{Arg}\,[-F(j\omega)] \tag{4}$$

so it is desirable to minimize $\theta_{-F}(\omega)$ at each ω .

If the above defined $n = 0$ for all $s \in \mathcal{N}$, then from (3B) $\theta_{-F}(\omega) <$ Arg $B(\omega) =$ Arg $P(j\omega) \triangleq \theta_P(\omega)$, $\forall\, \omega \in [0,\infty)$, giving $\inf \theta_{-F}(\omega) = \theta_P(\omega)$. Since F is mps, it is determined (up to an arbitrary constant factor) by θ_F , giving the supremum $F(s)$, denoted by

$$F_{sup} = -1/P_1(-s) = \frac{-(s+\lambda z)}{K(s+z)} \ .$$

K is obtained from: Numerator $(F_{sup}+P) = (s^2-\lambda^2 z^2)-kK(s^2-z^2) =$ $= s^2(1-kK)+z^2(kK-\lambda^2)$. The coefficients must have the same sign, giving $k_1K = K = 1$, $k_2K = \lambda^2 = k_2$. The root loci of $1+kP_1/F_{sup}$ are sketched in Figure 4. F_{sup} is obviously unique.

2.1.1 Case $n \neq 0$. Obviously from (4) it appears desirable to let $\theta_{-F} < \theta_P$ for some intervals, e.g., in $(0,\omega_x)$ in Figures 3a,b with $\theta_{-F} = \theta_P$ for

$\omega \in (\omega_x, \infty)$. The value of (4) would be presumably thereby increased. The trouble is that (3C) dictates $|F(j\omega_x)| > k_2 |P_1(j\omega_x)|$ with $|F(\infty)| < |P(\infty)| = 1$ as before, giving $k_2 < \left| \frac{1}{P_1(j\omega_x)} \frac{F(j\omega_x)}{F(\infty)} \right|$. But $|P_1(j\omega_x)|$ increases with ω_x , as does $|F(\infty)/F(j\omega_x)|_{min}$ as shown below. Thus (Horowitz 1963)

$$\ell_n \left| \frac{F(j\omega_x)}{F(\infty)} \right| = \int_0^\infty \mu(\omega)\ \theta_{-F}(\omega) d\omega \ , \quad \mu = \frac{2}{\pi}\ \frac{\omega}{(\omega_x^2 - \omega^2)} \ , \tag{5}$$

with $\mu(\omega) > 0$ for $\omega < \omega_x$, < 0 for $\omega > \omega_x$. Hence, to maximize (5), maximize θ_{-F} in $\omega < \omega_x$, minimize it in $\omega > \omega_x$. However, the set $\{F(j\omega)\}$ is constrained to have $\theta_{-F} < \theta_P$ for $\omega < \omega_x$, $\theta_{-F} > \theta_P$ in $\omega > \omega_x$, so let $\sup \theta_{-F} = \theta_P\ \forall\ \omega$. Thus (5) decreases vs ω_x , and $(\omega_x)_{opt} = 0$, giving the previous F_{sup} .

2.2 P(s) with General Uncertain Poles and Zeros.

In $P = k(s-z)/(s-p)$, $k \in [1, k_2]$, suppose z and p have independent uncertainty. Then a typical $T(\omega)$ is shown in Figure 3c for $z_{max} < p_{min}$. $B(\omega)$, the boundary of $T(\omega)$, has vertical lines due to the independence of the uncertainties. It should be clear from 2.1, 2.2 that the worst case determines max k_2 to be $\left(\frac{p_{min}}{z_{max}}\right)^2$. This is also seen by considering only the p_{min} , z_{max} case, for which $k_{2\ max}$ has the above value. The resulting design also obviously satisfies all other p , z smaller k_2 combinations. If $p_{min} < z_{max}$ and $p_{max} > z_{min}$, then nmp elimination is impossible for any finite k uncertainty, no matter how small.

Consider P as in above, but uncertain z, p, k related to some extent-, giving more general $T(\omega)$. The previous arguments leading to $-F_{sup}$ lying on $B(\omega)\ \forall\ \omega \in [0,\infty)$ still apply. The proof of existence and uniqueness of such an F_{sup} is basically the same as in (HOrowitz and Sidi 1978). However, it is more difficult now to find F_{sup}, because unlike the above $\operatorname{Arg} F_{sup}(\omega)$ is not apriori known. It is only known that there exists a unique mps analytic function which lies on $B(\omega)$ at each ω.

Finally, consider $P = k \dfrac{(s-z)}{(s-p)} P_2(s)$, P_2 having only ℓhp poles and zeros with uncertainty, e.g., $P_2(s) = \dfrac{(s+a_1)}{(s+a_2)(s+a_3)}$, $a_i \in [a_{i1}, a_{i2}]$. In order that the new $T(\omega)$ and $B(\omega)$ be qualitatively similar to the previous, we could let $F = \dfrac{F_1(s+a_o)}{(s+b_o)(s+c_o)}$, a_o fixed in $[a_1, a_2]$ etc. and then the problem is one of zeros of

$$F_1 + k \frac{(s-z)}{(s-p)} \frac{(s+a)}{(s+a_o)} \frac{(s+b_o)}{(s+b)} \frac{(s+c_o)}{(s+c)} \triangleq F_1 + P' .$$

Again, the reasoning in 2.1, 2.2 applies leading to the conclusion that F_{sup} lies on $B(\omega)$ at each $\omega \in [0,\infty)$, where $B(\omega)$ is that of P' over its range of uncertain parameters. In the balance of the paper, only uncertainty in k is considered, with generalizations applicable in the same manner as the above.

2.4 More Conservative Boundaries.

It is obvious that the above design philosophy applies to other kinds of bounds on the zeros of $F+P = \psi$. A reasonable alternative is that the zeros of ψ must have a real part $\leq -\sigma$, $\sigma > 0$. The line $\operatorname{Re} s = -\sigma$ replaces the imaginary axis, so let $s = w-\sigma$, $P(s) = k \dfrac{(s-z)}{(s-\lambda z)} = k \dfrac{[w-(\sigma+z)]}{w-(\sigma+\lambda z)}$ giving $\lambda_{new} = \dfrac{\sigma+\lambda z}{\sigma+z}$ and resulting new $k_{sup} = \lambda^2_{new} < \lambda^2$ and new

$$F_{sup} = \frac{w+(\sigma+\lambda z)}{w+(\sigma+z)} = \frac{s+2\sigma+\lambda z}{s+2\sigma+z} . \qquad (6)$$

Another possibility is that "damping factor of the zeros of ψ" $\geq \xi_o > 0$. The boundary is then a straight line in the second quadrant through the origin, at $\sin^{-1} \xi_o$ with the vertical. This is the new $\mathcal{N}$ but the procedure and results are as before, i.e., one finds $B(s)$, $s \in \mathcal{N}$, etc., and develops the integrals analogous to (4,5) in order to derive F_{sup}.

2.5 Reduction of Nonminimum-Phase Property.

In 2.1, 2.2 if $k_2 > \lambda^2$, then for some k, ψ of (2) must be nmp. However, one may design so that the 'worst' nmp < that in an uncompensated design. The concept of less or more of the nmp property is evident by noting that rhp zeros contribute excess phase lag (Horowitz 1963). The further the rhp zero from the origin, the less this excess lag vs ω, so if the nmp property cannot be eliminated, it is desirable to at least guarantee that the rhp zeros lie as far away as possible from the origin.

In this new problem, $\mathcal{N} = W_1W_2 \ldots W_5$, semicircle of radius R in Figure 5, because the forbidden region for the zeros of ψ is its interior. But the reasoning and technique is precisely the same as before. Again, the conclusion is that F_{opt} (not sup because the boundary of $\mathcal{N}$ is assumed permitted) lies on $B(s)$ for each $s \in \mathcal{N}$, and this corresponds to each $s \in \mathcal{N}$ on a root locus of $F_{opt} + kP_1$ for $k \in [1,k_2]$.

The derivation of F_{opt} is not difficult for the case of k uncertainty only. Arg P_1/F_{opt} must have a constant value on $\mathcal{N}$ (zero or 180°, the former here). The inverse image with respect to $W_1 W_2 W_3$, of the zero-pole pair at z, λz is a pole-zero pair at $R^2/\lambda z$, R^2/z, i.e., this combination

of 2 pole-zero pairs has a constant angle on $W_1 W_2 W_3$. The inverse image of these two pairs with respect to W_5 is their ℓhp image, shown in Figure 5, with the resulting root loci. The new k_{2max} is now

$$\frac{\lambda^2(R^2-z^2)^2}{(R^2-\lambda^2 z^2)^2} > \lambda^2 \tag{7}$$

e.g., if $z = 2$, $\lambda = 4$, $R = 10$, then $\frac{\text{new } k_{2max}}{\text{old } k_{2max}} = \frac{64}{9}$. Here F_{opt} has rhp pole and zero, apparently violating the constraint that F is mp and stable. This is because the actual constraint used is that ℓnF be regular in the forbidden region, which is now the interior of $W_1 W_2 \ldots W_5$.

Of course, one may define different shapes for $\mathcal{N}$, e.g., in a specific problem, a highly underdamped rhp zero pair may be more tolerable closer to the origin than a more damped pair further from the origin (Horowitz 1963). An ellipse with horizontal major axis might then be more suitable for $\mathcal{N}$. The conclusion that F_{opt} lies on $B(s)$ for $s \in \mathcal{N}$ probably still applies, but it may be more difficult to find F_{opt}, or a good approximation to it.

3. MORE COMPLEX P(s)

While the results in Section 2 are of interest in themselves, they are more important in suggesting an approach usable for more complex P . An important conclusion was that $-F_{sup}(\omega) \in B(\omega)$, $\forall$ $\omega \in [0,\infty)$ which coincided with $\theta_{-F} = \theta_P$. The more complex problems next treated show that in most cases $\theta_{-F} = \theta_P$ does not coincide with $-F_{sup} \in B(\omega)\ \forall\ \omega$. Each case considered introduces a new property.

3.1 Case 1 - High Frequency Modification.

Consider $P = \dfrac{-k(s-p)}{s^2-Es+H} \triangleq k\,P_1(s)$, $k \in [1,k_2]$ with $p,E,H > 0$, and such that $\theta_P \in (-.5\pi,0)$ in $(0,\omega_*)$, $\in (0,.5\pi)$ in (ω_*,∞) and $|P(j\omega)| > P(0)$, as in Figures 6a, b. If $k_2 < OQ/OA$ in Figure 6a, then a constant value for $-F$ with $k_2P_1(0) < -F < OQ = P_1(j\omega_*)$ is satisfactory. But for larger k_2 a dynamic $F(s)$ is needed, such that $-F$ encircles $\{B(\omega)\}$ once negatively. Given $-F(0) > k_2P_1(0)$, this requires the existence of an ω_x at which $|F(j\omega_x)| < |P_1(j\omega_x)|$, $\theta_{-F}(\omega_x) = \theta_P(\omega_x)$, and $-F(\infty) > k_2P_1(\infty) = 0$. (It is easily seen that $-F(0) < P_1(0)$ is inconsistent with mps F .) Also in Figure 6b, $-F$ must <u>in the net</u> execute the passage from above $B(0)$ to below $B(\omega_x)$ on the right of $B(\omega)$ $(\theta_{-F} > \theta_P)$, and the passage from below $B(\omega_x)$ to above $B(\infty)$ <u>in the net</u> on the left of $B(\omega)$ with $\theta_{-F} < \theta_P$ (cf 2.1 after (3) and Figures 3a, b). An acceptable $-F$ is shown in Figure 6b with OK1 , OK2 two among many possibilities for $\omega > \omega_x$.

The condition on F at ∞ is easy to achieve, so k_2 maximization involves maximization of

$$\ell n \left|F(0)/F(j\omega_x)\right| = \frac{2\omega_x^2}{\pi}\int_0^\infty \mu(\omega)\theta_{-F}d\omega \quad , \; \mu(\omega) = \frac{1}{\omega(\omega^2-\omega_x^2)} \tag{8}$$

< 0 for $\omega < \omega_x$, > 0 for $\omega > \omega_x$. Hence, it is desirable to minimize θ_{-F} in $\omega < \omega_x$ and maximize it in $\omega > \omega_x$. However, it was noted above that in the net $\theta_{-F} > \theta_P$ in $(0,\omega_x)$.

One might suggest use of $\theta_{-F} < \theta_P$ for some intervals, i.e. $n \neq 0$ as in 2.1.1, but a technique similar to that of Equation (5) can be used to prove it worse here also. Thus it would require existence of an $\omega_o < \omega_x$ at which $\theta_{-F} = \theta_P$ and $|F| > k_2|P_1|$. Maximization of

$$\ell n \left|F(j\omega_o)/F(j\omega_x)\right| = \frac{2}{\pi}\int_0^\infty \frac{\omega\theta_{-F}(\omega_x^2-\omega_o^2)d\omega}{(\omega_x^2-\omega^2)(\omega_o^2-\omega^2)} \tag{9}$$

derivable from (8) by considering $\ell n\left|\frac{F(j\omega_x)}{F(0)}\right|$ and $\ell n\left|\frac{F(j\omega_o)}{F(0)}\right|$ leads, as in 2.1.1, to $\omega_o = 0$.

The above suggests trying $\theta_{-F} = \theta_P$, i.e., $-F = -F_1 = \frac{s^2+Es+H}{K(s+p)}$, qualitatively shown in Figure 6a. It is interesting to relate this to the root loci of $1+P/F_1$ shown qualitatively in Figure 6c. The numbers $E = 5$, $F = 6$, $p = 1$ are taken for illustration. The double root at $j\omega_x = j3.9$ corresponds to M (on P_{min}) — note that $-F_1$ lies on $B(\omega)$ on both sides of M in Figure 6a and to the lowest k for which the root loci are all on the $j\omega$ axis in Figure 6c. The double root

at the origin corresponds to V in Figure 6a — with a similar property of $-F$ when the locus for $\omega < 0$ is included. The single root at $j4.8 = j\omega_y$ (at k_2) corresponds to point Y in Figure 6a — at which $-F$ lies on $B(\omega)$ only for $\omega < \omega_y$.

Since $-F_1$ lies on $B(\omega)$ for $\omega < \omega_x$, no improvement (decrease of θ_{-F}) is possible there. But it is possible to improve matters by increasing θ_{-F} for $\omega > \omega_y$ because $-F_1$ does not lie on the $B(\omega)$ for $\omega > \omega_y$. Any such increase will however decrease $|F/F(0)|$ $\forall$ $0 < \omega < \lambda\omega_y$ for some $\lambda > 1$ (Horowitz 1963), creating new ω_x, ω_y and larger k_2. A simple way to increase θ_{-F} for $\omega > \omega_z > \omega_y$ is to let $-F_2 = \dfrac{(s^2+Es+H)(s^2+\omega_z^2)}{K(s+p)}$, which leaves θ_{-F} unaltered in $(0,\omega_z)$. It is relatively simple to find ω_z (= 10.9) from Num. (F_2+P) by requiring that at $k_1 = 1$, it has a factor $(s^2+\omega_1^2)^2$; $\omega_1 = 2.1$ replaces ω_x in Figures 6b,c. At k_2 it should have a factor $s^2(s^2+\omega_2^2)$ giving $\omega_2 = 7.24$ replacing ω_y. (Compare Figures 6a,c at $\omega = 3.9, 7.24, 9.8, 10.9$). $-F_{2n}$ is shown in Figure 6a and permits a slightly larger $k_2 = 1.797$ in place of the 1.731 for F_1. Note that $-F_2$ lies on $B(\omega)$ for $\omega \in (0,9.8) > (0,4.8)$ for which $-F_1$ lay on $B(\omega)$.

One may increase k_2 by repeating the above with the factors

$$\left(\frac{s^2+A_1}{s^2+B_1}\right)\left(\frac{s^2+A_2}{s^2+B_2}\right) \cdots \left(\frac{s^2+A_m}{s^2+B_m}\right)(s^2+A_{m+1})$$

$A_i < B_i < A_{i+1}$. Such increases are always possible if $\exists$ any interval in which $-F$ does not lie on $B(\omega)$. However, the increase obtained is found to be exceedingly small and does not justify the design effort.

3.2 Case Two - Low Frequency Modification.

Let $P = k(s^2-As+B)/(s^2-Cs+D) \triangleq k\,P_1$, $k \in \{1,k_2\}$ with parameters such that P is as shown in Figure 7a, i.e., $.5\pi > \theta_P > 0$ in $(0,\omega_1)$, $-.5\pi < \theta_P < 0$ in (ω_1,∞) ; $|P(j\omega_1)| < |P(\infty)| < P(0)$. Again, a constant $-F$ can handle uncertainty $k_2 < |P_1(j\omega_1)|^{-1}$, if $k_2|P_1(j\omega_1)| < -F < 1$. For larger k_2 a dynamic $F(s)$ is needed satisfying the following conditions

$$\left.\begin{aligned} &-F(0) < P_1(0) \ ; \ F(\infty) < P_1(\infty) \\ &\exists\, \omega_x \ni \theta_P = \theta_{-F} \text{ but } -F(j\omega_x) > k_2\, P(j\omega_x) \end{aligned}\right\} \tag{10}$$

$F(s) \neq k\,P_1(s)$, $s \in \mathcal{N}$ and $\mathcal{E}_\psi = -1$.

From (10) it is desirable to maximize $|F(j\omega_x)|/|F(0)|$, $|F(j\omega_x)/F(\infty)$. The effect of θ_{-F} on the former is given by (8). Its effect on the latter is (Horowitz 1963)

$$\ell n \left|\frac{F(\omega_x)}{F(\infty)}\right| = \frac{2}{\pi}\omega_x^2 \int_0^\infty \theta_{-F}(\omega)\lambda(\omega)d\omega \ , \ \lambda(\omega) = \frac{\omega}{\omega_x^2-\omega^2} \ . \tag{11}$$

Thus, from the signs of the weighting functions $\mu(\omega)$, $\lambda(\omega)$ in (8.11), it is desirable to minimize $\theta_{-F} \in (0,\omega_x)$ and maximize it in (ω_x,∞) subject of course to (10). However, from previous experience one might guess, and it is in fact so, that in these intervals inf and sup θ_{-F} are equal to θ_P , so it is reasonable to try $(s^2+Cs+D)/(s^2+As+B) \triangleq -F_1$ -- Figure 7a, giving $k_2 = \frac{OB}{OA}$. It is seen that ω_x of (10) = ω_1 and that indeed θ_P constrains θ_{-F} but only in (ω_b,∞) because $-F_1$ lies on $B(\omega)$ only in (ω_b,∞) . Note that $n=0$ is assumed. Proof that $n\neq 0$ is inferior, is similar to that previously given in 2.1.1, 3.1.

Accordingly, θ_{-F} may be increased in $(0,\omega_b')$ (because the change changes ω_b), but in such a manner that $\theta_{-F} = \theta_P$ in (ω_b',∞). A simple way to do this is to let $F_2 = F_1 \frac{s^2}{(s^2+\omega_2^2)}$, giving $\theta_{-F} = \theta_P$ for $\omega > \omega_2$ but $\theta_{-F} = \theta_P^2$ for $\omega < \omega_2$. The value of ω_2 is found by considering the numerator of

$$1 + \frac{P}{F} = 1 - \frac{k(s^2-As+B)(s^2+As+B)(s^2+\omega_2^2)}{s^2(s^2-Cs+D)(s^2+Cs+D)}$$

whose (qualitative) root loci are shown in Figure 7b, for the case $P = \frac{k(s-1)(s-2)}{(s-.5)(s-3)}$. One chooses ω_2 so that at $k=1$, the numerator has two zeros at ∞ and a factor $(s^2+\omega_o^2)^2$. Note that $-F_1$ lies on $B(\omega)$ only for $\omega > \omega_b = .64$ in this example, while $-F_2$ lies on $B(\omega)$ for $\omega > \omega_a = .3$. Note the relations between ω_o, ω_a, ω_c and the k values in Figures 7a,b — cf similar comparison of Figures 6a, 6c. The difference in k_2 is however, not large — 1.41 = OB'/OA instead of 1.39 = OB/OA in Figure 7a. Increased improvement is, of course, possible because $-F_2$ does not lie on $B(\omega) \in (0,.3)$, but it is already seen that the improvement will be very slight — for example use of $(s^2+\omega_b^2)^2/s^4$ gives $k_2 = 1.412$ with $-F_3$ on $B(\omega) \in (0,.138)$, $(.287,\infty) > (.3,\infty)$ for $-F_2$.

3.3 Case Three - High Frequency Modification.

In this case P has a pair each of rhp poles and zeros $\ni 0 < \theta_P < .5\pi$ in $(0,\omega_o)$, $-.5\pi < \theta_P < 0$ in (ω_o,∞), $|P(j\omega_o)| < P(0) < P(\infty)$, e.g., $P = k\frac{(s^2-s+2)}{(s^2-3s+4)}$, $k \in [1,k_2]$ in Figure 8a. Using $-F_1 = \frac{K(s^2+3s+4)}{(s^2+s+2)}$ gives $K = 4$, $k_2 = 2.88$ and $-F$ on $B(\omega)$ for $\omega \in (0,2.08)$. It is therefore

possible to improve θ_{-F} in the high-frequency range. Here, the constraints on a realistic $-F$ are: $F(0) < P(0)$ and at some ω_1, $\theta_{-F} = \theta_P$ with $|F(j\omega_1)| > k_2|P(j\omega_1)|$, so maximization of $|F(j\omega_1)/F(0)|$ is desirable. From (8) this is achieved by maximizing θ_{-F} in $(0,\omega_1)$ and minimizing it in (ω_1,∞). But such a $-F$ will be on $B(\omega)$ in $(0,\omega_1)$, so the improvement can be made only in (ω_1,∞). This is conveniently done by means of poles at $\pm j\omega_1$, which leaves $\theta_{-F} = \theta_P$ in $(0,\omega_1)$. The resulting root loci of $1 + P/F_2$ are then shown qualitatively in Figure 8b, for the above numerical example. To optimize, ω_1 is chosen so that $k_{11} = k_{12} = k_1 = 1$. The result is $k_2 = 3.23 >$ previous 2.88 with $\omega_1 = 3.987$, and $-F_2$ lies on $B(\omega)$ for $\omega \in (0,3.76)$, cf previous $(0,2.08)$. In this case use of two coincident complex pole pairs in place of the single pair leads to small increase of k_2 (3.27 in place of 3.23 with $-F_3$ on $B(\omega) \in (0,4.42), (5.84,6.43)$. Additional improvement is, of course, still possible because $-F$ does not lie in $B(\omega)$, for all ω.

3.4 Case Four - Medium and High Frequency Modification.

In this case, P has three rhp poles and two rhp zeros with coefficients such that $P_{\mathcal{N}}$ is as shown in Figure 9a, i.e., in the first quadrant for some $(0,\omega_1)$, in the fourth for (ω_1,ω_2) and back in the first for (ω_2,∞), with $|P(j\omega_2)| > |P(0)| > |P(j\omega_1)|$ and $P(\infty) = 0$, e.g.,

$P = k\,P_1 = \dfrac{-k(s-.8)^2}{(s-.2)(s-6)(s-10)}$, $k \in [1,k_2]$. Since a constant $-F$ is capable of handling some range of k, it is reasonable to try $-F_1 = \dfrac{K(s+.2)(s+6)(s+10)}{(s+.8)^2}$, choosing K to maximize k_2; giving $k_2 = 4.36$, $K = 351.6$. The root loci of

$(1+P/F)$ are shown in Figure 9b (qualitative). Note that $-F_1$ does not lie on $B(\omega)$ in $(4.0, 14.1)$ and $(37.4, \infty)$ and the relations between these points in Figures 9a,b. Hence, θ_{-F} can be changed in these intervals, to permit larger k_2.

How should θ_{-F} be modified in these two intervals? In this case $\mathcal{E}_\psi = -1.5$ is required, so there must exist $\omega_x, \omega_y > \omega_x$, $\ni F(0) < P_1(0)$, $|F(j\omega_x)| > k_2|P_1(j\omega_x)|$, $|F(j\omega_y)| < |P_1(j\omega_y)|$, $F(\infty) > 0$ with $\theta_{-F} = \theta_P$ at ω_x, ω_y. (For F_1, $\omega_x = .75$, $\omega_y = 4$. There is another ω_x, ω_y pair at $(14.1, 37.4)$ because $F(\infty) = \infty$ but they need not be considered.) In Figure 9a, it is seen that $|F(j\omega_x)/F(0)|$ is the constraining factor, so the freedom in θ_{-F} in the two intervals should be used to maximize this factor. From the sign of $\mu(\omega)$ in (8), this means decreasing θ_{-F} in these two intervals, because $\omega_x < 4$. In the first interval, this can be done by letting $F_2 = K_2F_1(s^2+B)/(s^2+A)$, $A < B$ and choosing K_2, B, A to maximize k_2. When this is done an improvement is indeed found, but it is very small (4.375) and the region with $-F$ on $B(\omega)$ increases to $\sim (0, 8.9)$ and $(27.8, 58.1)$. In the second interval the modification to F would logically consist of a factor (s^2+B) in the denominator. This leads to $-F_3$ on $B(\omega)$ in $(0, 10.6)$ and $k_2 = 4.379$, with the root loci in Figure 9c, for this modification only. One could use both modifications but in view of the very small k_2 increases for F_2, F_3 the effort is not worthwhile.

4. CASE CONSTANT F NOT SATISFACTORY

In all previous problem classes there existed a finite k range for which $-F$ a constant was satisfactory. This was due to P curving sufficiently such that it encircled negatively some point on the real axis $N/2$ times, with N the number of rhp poles of P. Our technique was based on this assumption, as one could then start with $\theta_{-F_1} = \theta_P$ giving $-F_1$ the $j\omega$ axis image of P. The resulting F_1 could be improved if there existed any ω interval in which $-F_1$ did not lie on the $B(\omega)$. This is not always the case, e.g., $P = \dfrac{k(s-1)(s-2)}{(s-3)(s-4)}$ has $\theta_P \leq 0 \; \forall \; \omega \in [0,\infty)$, giving $\mathcal{E}_\psi = -.5$ at most for $-F$ a constant. Use of $\theta_{-F} = \theta_P$ no longer gives a satisfactory design for any finite k range, no matter how small. It is shown below how any such P can be modified to a new P which does have the desired $\mathcal{E}_\psi = -N/2$ for $-F$ a constant. One can then find F_{sup} for this new P as before. Furthermore, optimization of the new P is also possible, as illustrated later. The modification of P involves insertion of additional rhp zeros and/or poles.

Let $P = \dfrac{n}{d} P_{old} = \dfrac{n n_o}{d d_o}$, be the modified P and $T = \dfrac{P}{1+P} = \dfrac{n n_o}{d d_o + n n_o} = \dfrac{n n_o}{D}$ where the n_i, d_i are polynomials with real coefficients. The desired $N/2$ encirclement condition for P is equivalent to D having no rhp zeros. If D has only interior ℓhp zeros then continuity of the zeros of D with respect to its parameters, guarantees the existence of a finite range of each for which D has only ℓhp zeros. At each rhp zero of d_o (denoted by z_i, $i = 0,\dots,m$), $T = 1$ and $n(s) = \sum_0^m A_i s^i$ is chosen to achieve this as follows.

Let $D(s)$ be a real Hurwitz polynomial of degree $\geq$, larger of [degree of $d_o(s)$, or of $n\,n_o(s)$] , none of whose zeros are those of $n(s)$ but is otherwise arbitrary. Setting $n(z_i)n_o(z_i) = D(z_i)$ gives $m+1$ linear equations in the $m+1$ A_i coefficients. As the resulting matrix in the Vandermondian, there is a solution, giving $n(s)$. Then $d(s)$ is obtained from $d\,d_o = D - n\,n_o$, in which D , n_o , d_o , n are known. In the above the rhp zeros of d_o were assumed distinct. One can extend the proof to the general case but it is simpler to suggest that one can always perturb the multiple root case to distinct roots with negligible effect on the problem. The above procedure is certainly practical but one may decrease the degrees of $n(s)$, $d(s)$ by a suitable rather than arbitrary choice of $D(s)$, as shown in a later example.

The above constitutes proof of the following theorem. Its generalization to non-real P , G and to an arbitrary half plane is obvious. It is equally obvious that it may be extended to apply to nonzero independent variations in the poles and zeros of P , as well as to its gain factor (cf 2.).

<u>Theorem</u>: Given $P(s) = k\,\dfrac{n_o(s)}{d_o(s)}$, with n_o , d_o real polynomials with arbitrary zeros. It is always possible to find a real rational function $G(s)$, such that $1+P(s)G(s)$ has all its zeros in the interior of the ℓhp for a nonzero range of k .

<u>Example.</u> $P = \dfrac{k(s-1)(s-2)}{(s-3)(s-4)}$, for which constant F is no solution as $P(j\omega)$ is in the fourth quadrant $\forall\ \omega \in (0,\infty)$. Using the previous notation, try first $T = \dfrac{k(s-1)(s-2)}{s^2+As+B}$ and see if $A,B > 0$ can be chosen $\ni$ $T(3) = T(4) = 1$.

This is not possible. It is also found that in $T = \frac{K(s-1)(s-2)}{(s^2+As+B)(s+C)}$ it is impossible to find $A,B,C > 0 \ \ni \ T(3) = T(4) = 1$. However, it can be done with $T = \frac{k(s-1)(s-2)(s+p)}{(s^2+As+B)}$; e.g., $A = B = 1$, $p = -31/6$, $K = -3$.

Thus for $P_{new} = P^* = \frac{k(s-1)(s-2)(s-\frac{31}{6})}{(s-\frac{5}{6})(s-3)(s-4)}$ there exists a constant F , $\ni$ $F+P^*$ is Hurwitz for a finite range of k .

Optimization is achieved by first letting $\theta_{-F} = \theta_{P^*}$ giving

$F = \frac{K_1(s+\frac{5}{6})(s+3)(s+4)}{(s+1)(s+2)(s+\frac{31}{6})}$ so that the root loci of $1 - P^*/F$ are as shown in Figure 10 with $k_1 = k_{min}$ = the larger of k_{11}, k_{12} and k_2 the smaller of $(1, k_{21})$. An optimum design would be achieved if $k_{11} = k_{12}$ and $k_{21} = 1$ (giving roots at $\pm j\infty$) , for then $-F(j\omega)$ lies on $B(\omega)$ $\forall$ $\omega \in [0,\infty)$.

We therefore write $P^* = \frac{k(s-1)(s-2)(s-z)}{(s-p)(s-3)(s-4)}$, $F = \frac{(s+p)(s+3)(s+4)}{(s+1)(s+2)(s+z)}$ and seek z , p to achieve the two conditions $k_{11} = k_{12}$, $k_{21} = 1$. The result is $z = 5.275$, $p = .832$, giving $k_2/k_1 = 1.117$ (in place of 1.068 for $z = 31/6$, $p = 5/6$) and $-F$ on $B(\omega)$ $\forall$ $\omega \in [0,\infty)$.

5. CONCLUSIONS

A design philosophy and methodology has been presented which is useful for eliminating or reducing the nmp property for structures of the form of Figure 1. One first deals with a P function, such that F a constant guarantees the Hurwitz character of the numerator of $1+P/F$, over some nonzero range of the gain factor k of P. It was shown in 4. that the original P can always be modified to have this property. Then $\theta_{-F_1} = \theta_P$ gives a first attempt F_1, which gives a larger tolerable range of k than F a constant. If the resulting $-F_1$ does not lie on $B(\omega)\ \forall\ \omega$, then F can be improved. The means of improvement lies in deciding whether additional phase lag or lead of F is desirable in those intervals where $-F_1$ does not lie on $B(\omega)$. The Bode integrals are used in making this decision and the implementation is by means of poles and zeros on the $j\omega$ axis, such that θ_F is unchanged in the other intervals.

However, the optimization of F was always over the mps class, and it is conceivable that non-mps F may be superior. One way to handle this possibility in the context of this paper, is to imbed P in a larger nmp unstable class with more rhp poles or/and zeros than the original and apply the approach of this paper. These additional rhp critical points are of course supplied by F. Thus, direct optimization on the most general class of F is not presented in this paper.

Uncertainty in the poles and zeros of P is easiest to handle when these uncertainties are independent. For then, there exist extreme pole-zero combinations which limit the maximum range of k and give the resulting F . The latter is then satisfactory for all the other pole-zero possibilities. These extreme pole-zero combinations may be such that no F is at all available. The problem of finding F_{sup} can be much more difficult if the uncertainties are related.

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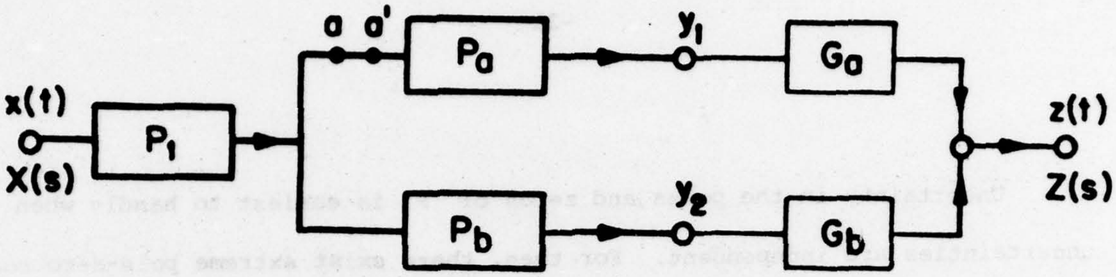
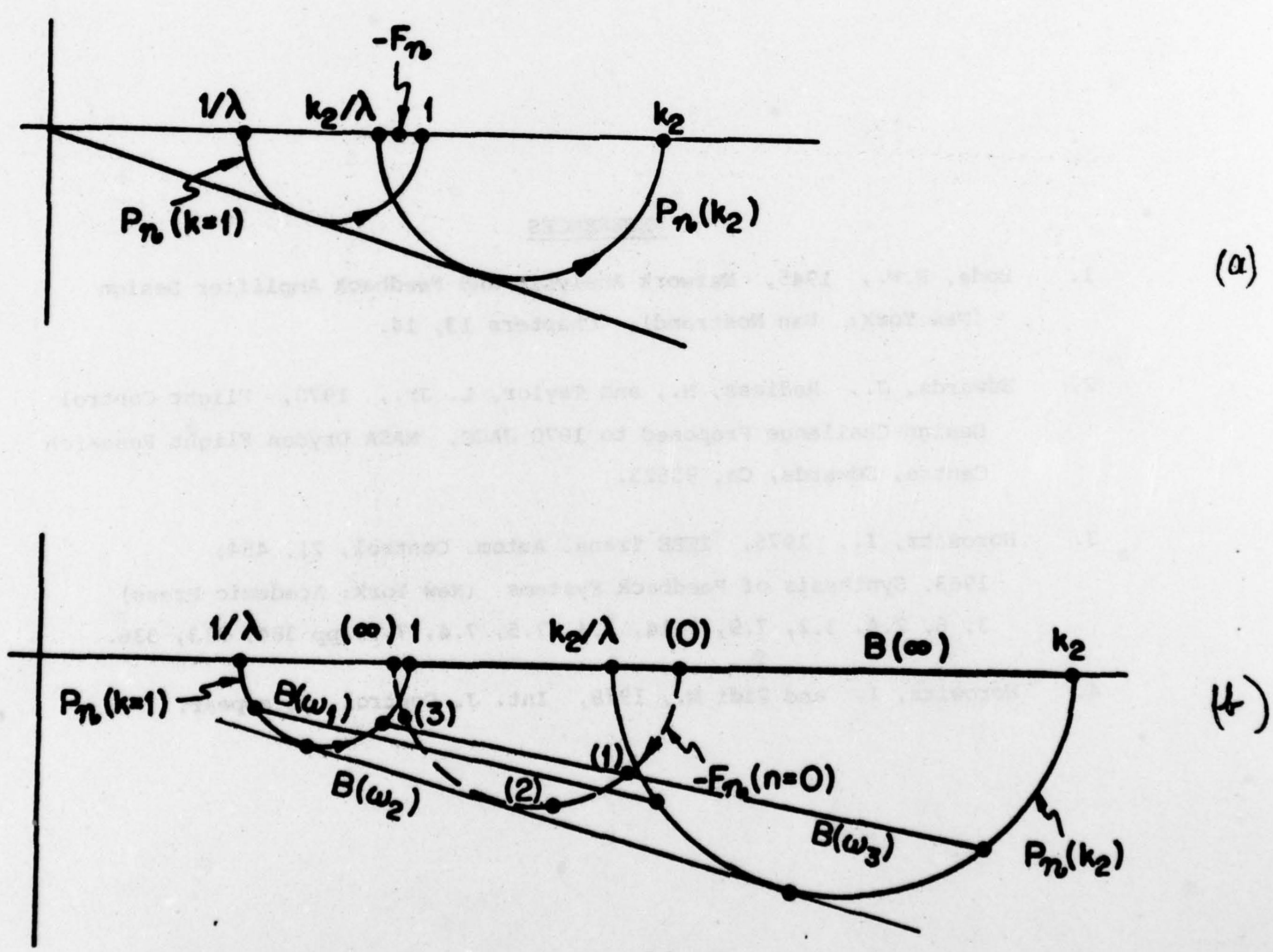


1. **Plant for which nmp alleviation is possible.**

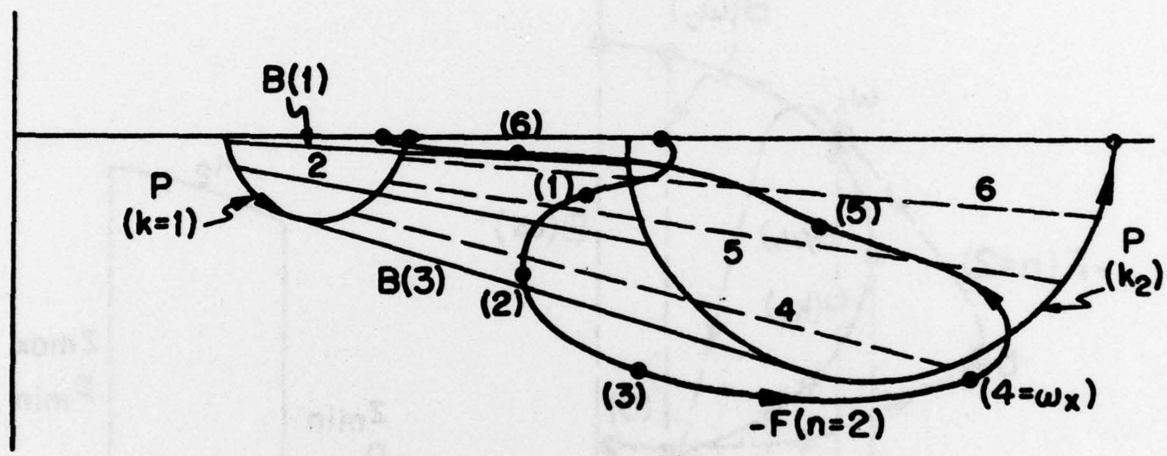


2. **Case 1. Nyquist locus of** $P = \frac{k(s-z)}{(s-\lambda z)}$, $k \in [1,k_2]$,

(a) $1 < k_2 < \lambda$, **constant F useable,**

(b) $k_2 > \lambda > 1$, **dynamic F needed.**

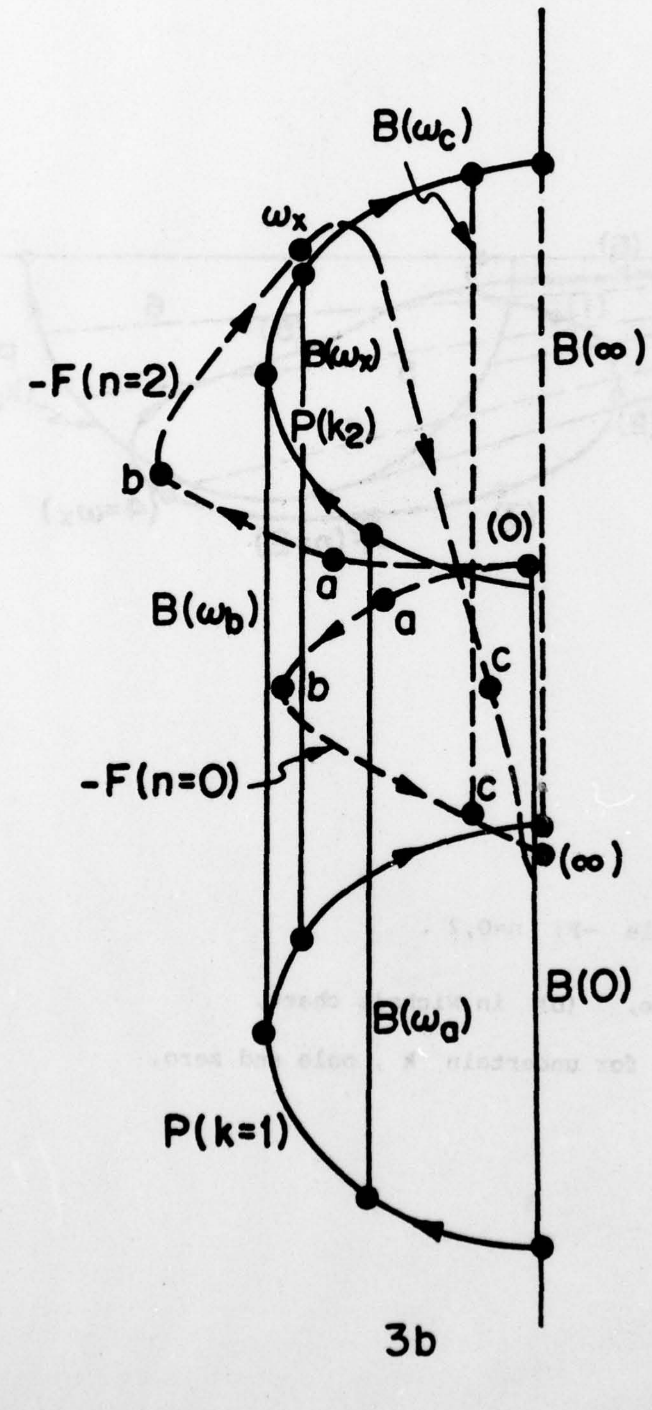
(3a)



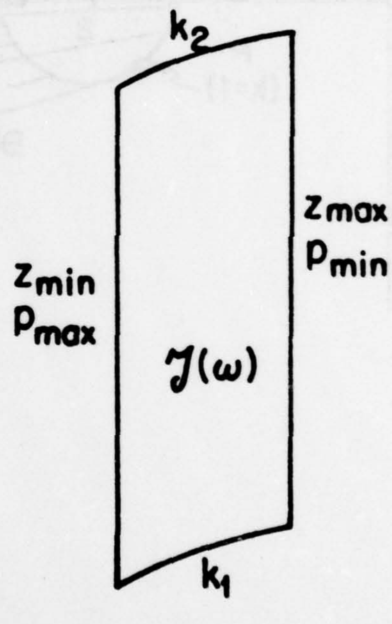
3. Examples of acceptable -F: n=0,2 .

(a) in complex plane, (b) in Nichols chart,

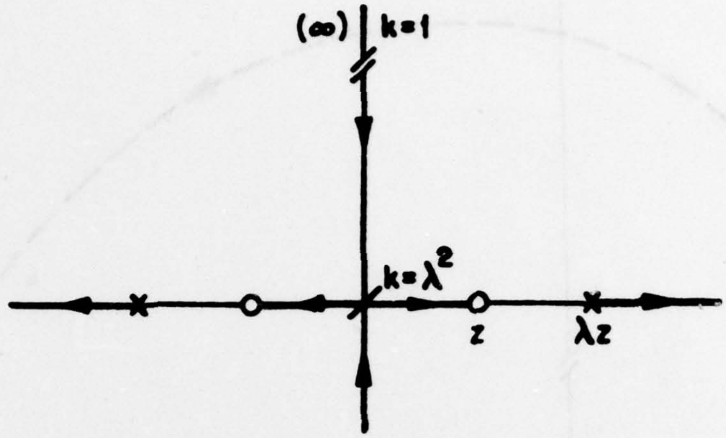
(c) Template of P for uncertain k , pole and zero.


B(ωc)
ωx
B(ωx)
B(∞)
-F(n=2)
P(k2)
b
(O)
a
B(ωb)
a
c
b
-F(n=O)
c
(∞)
B(O)
B(ωa)
P(k=1)

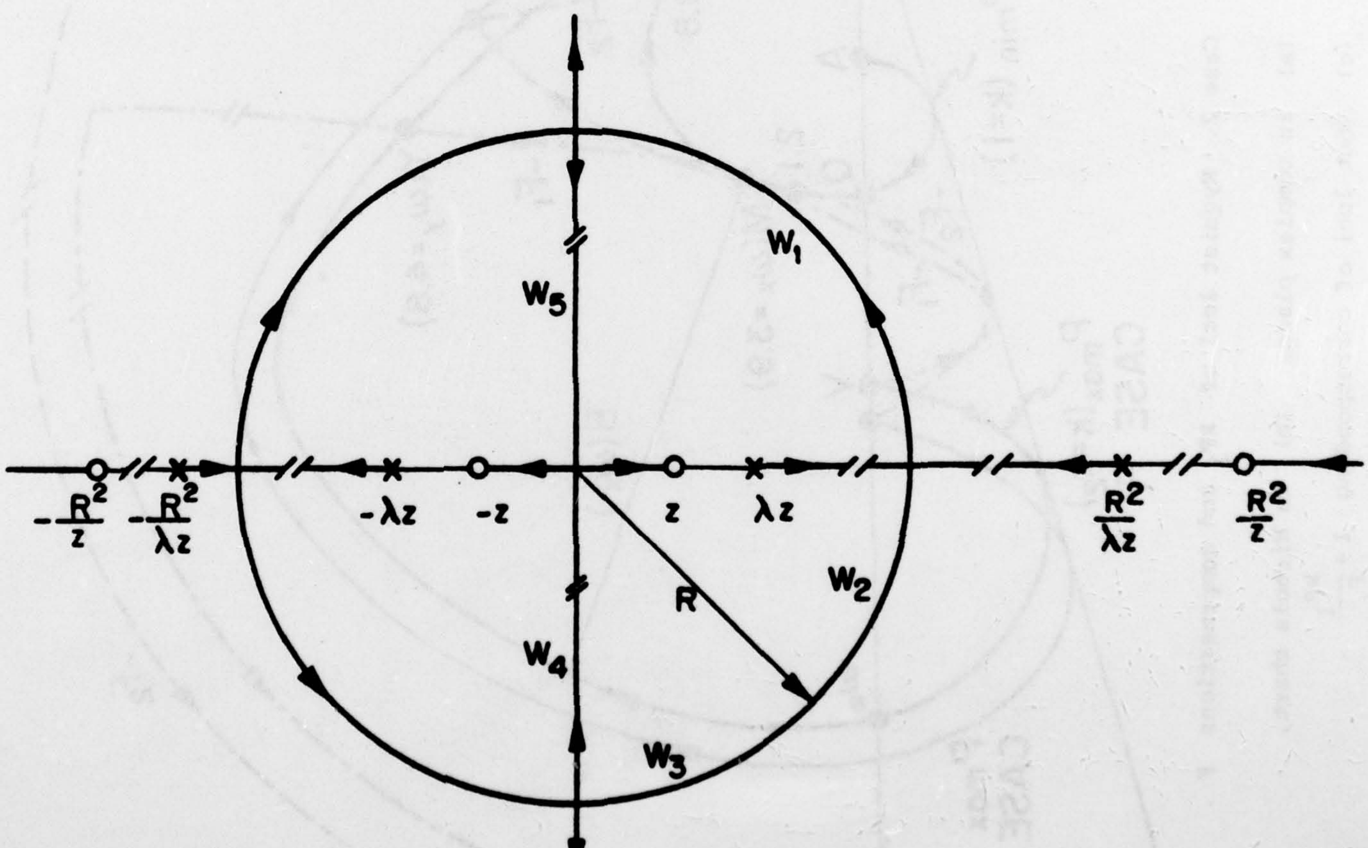
3b


k2
Zmax
Pmin
Zmin
Pmax
J(ω)
k1

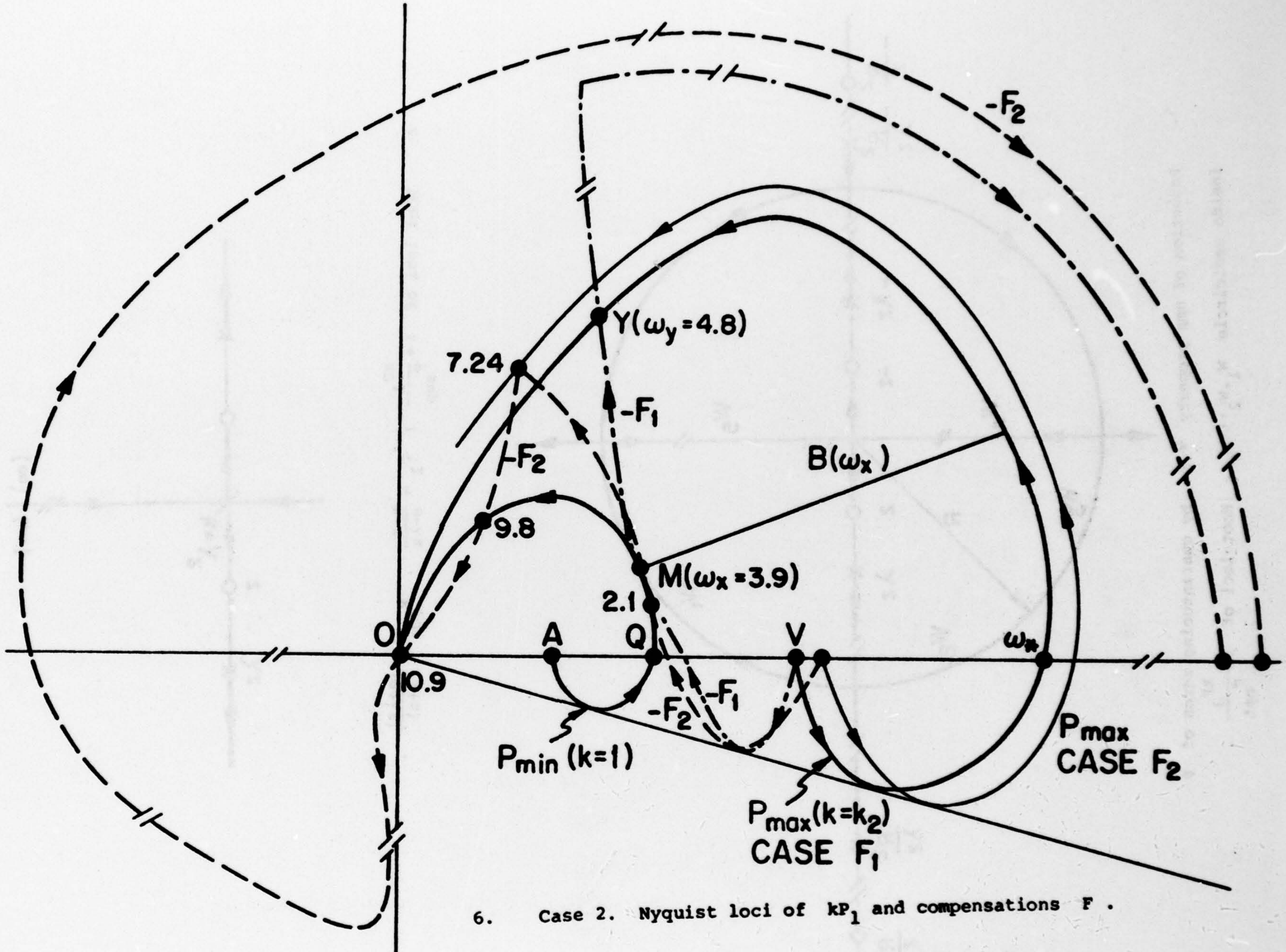
3c



4. Root loci of $1+\frac{kP_1}{F_{sup}}$, $P_1 = \frac{s-z}{s-\lambda z}$, $F_{sup} = \frac{(s+\lambda z)}{(s+z)}$.



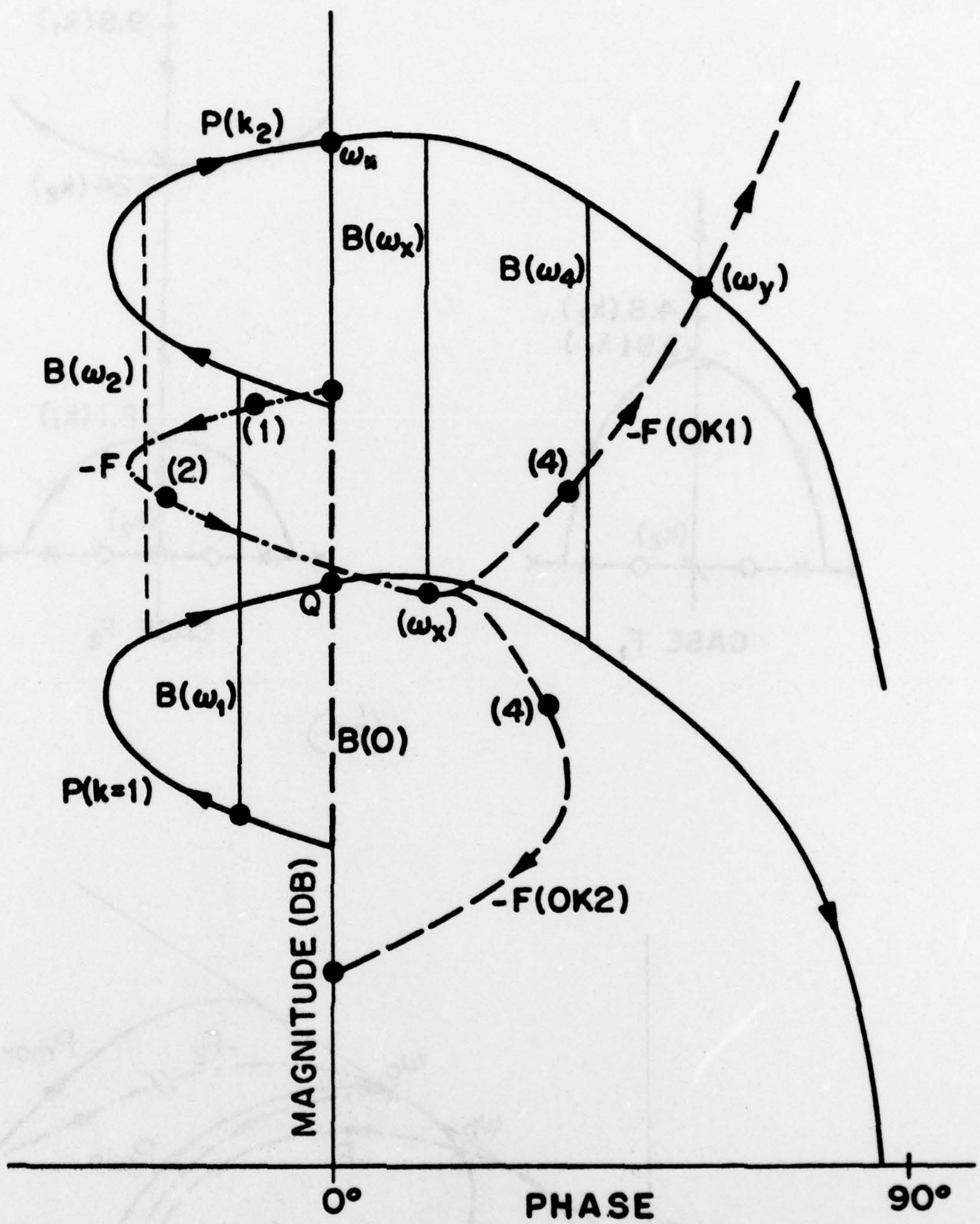
5. Reduction of nmp property ∀ k , by guaranteeing zeros of ψ inside semicircle $W_1, W_2 \cdots W_5$. Root loci of $1+\frac{kP_1}{F_{opt}}$.



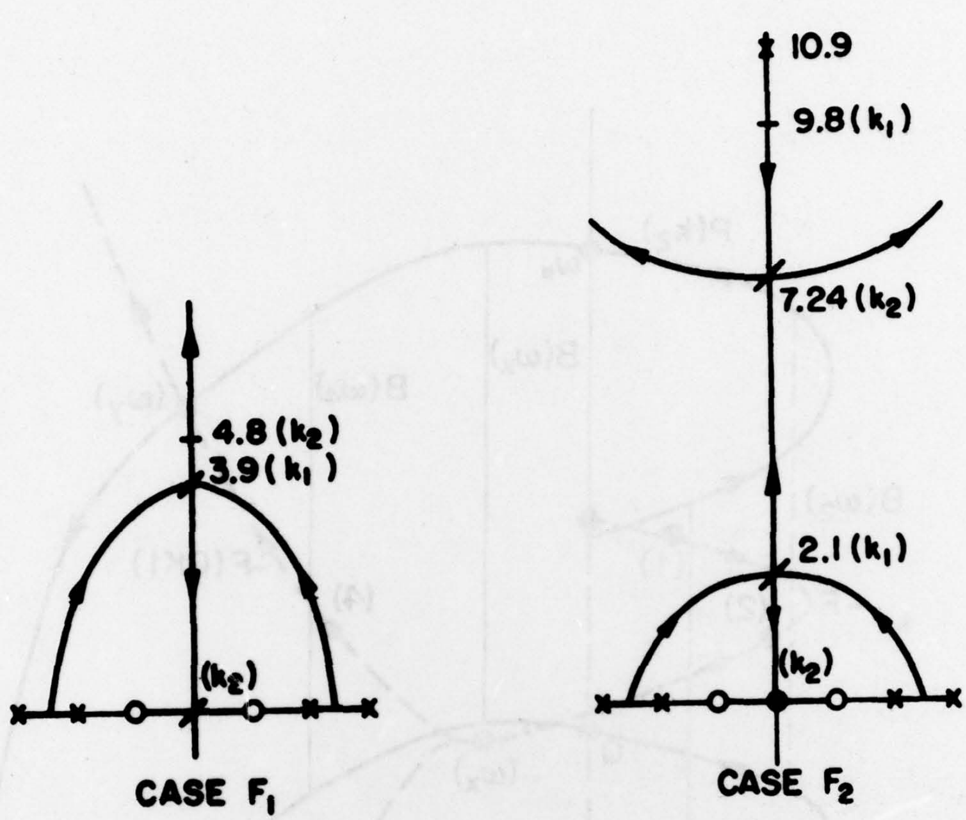
6. Case 2. Nyquist loci of kP_1 and compensations F.

(a) in complex plane, (b) in Nichols chart,

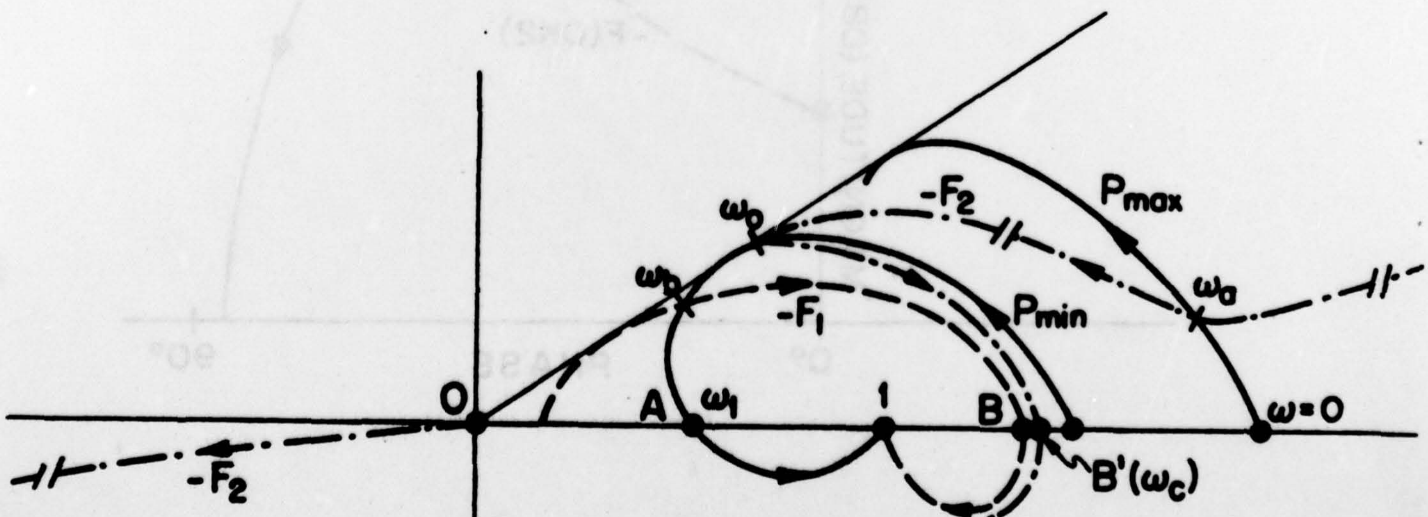
(c) Root loci of corresponding $1 + \frac{kP_1}{F_i}$.


P(k2)
ωx
B(ωx)
B(ω4)
(ωy)
B(ω2)
(1)
-F(OK1)
-F
(2)
(4)
Q
(ωx)
B(ω1)
(4)
B(0)
P(k=1)
-F(OK2)
MAGNITUDE (DB)
0°
PHASE
90°

(6b)

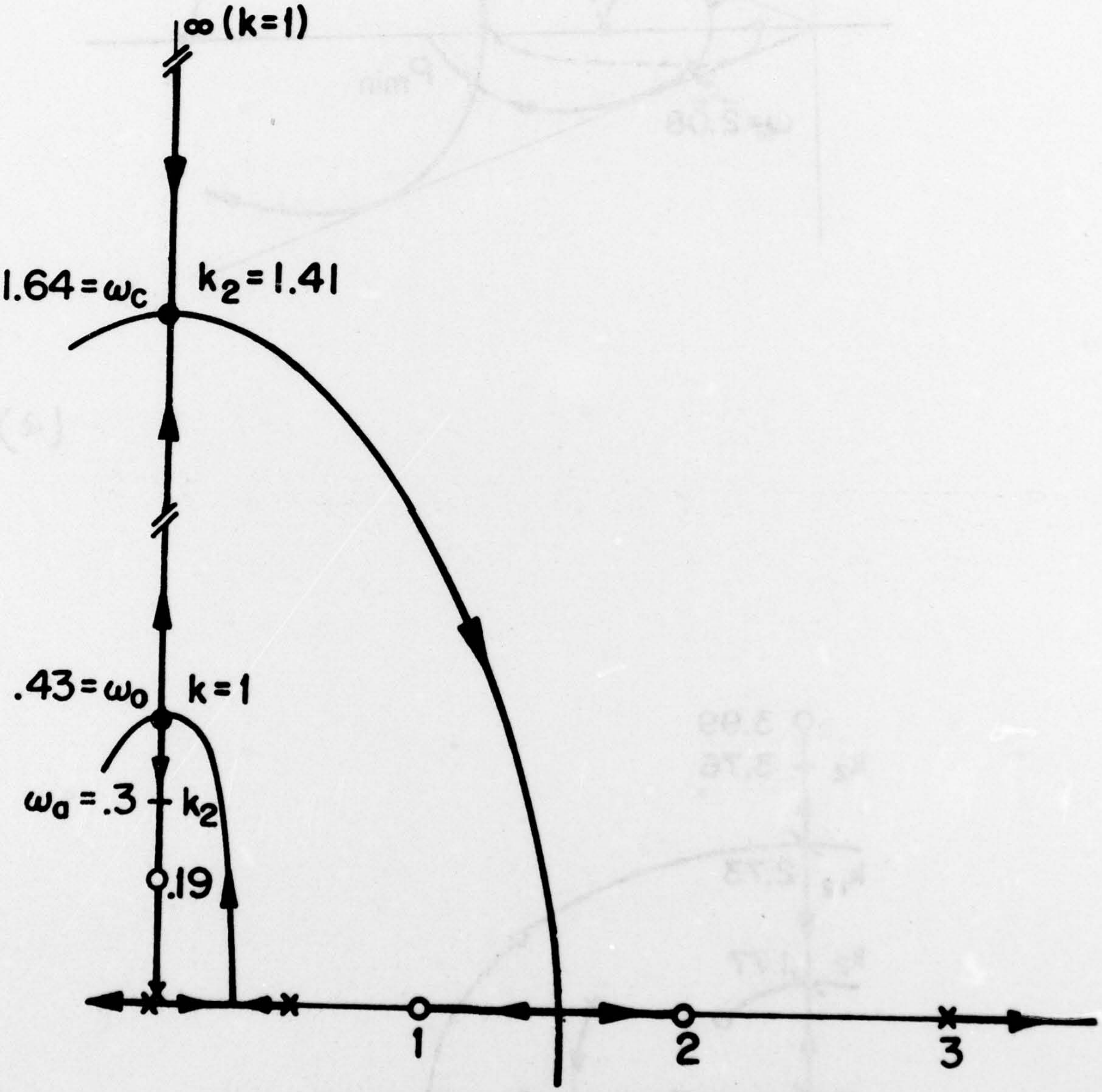


(bc)

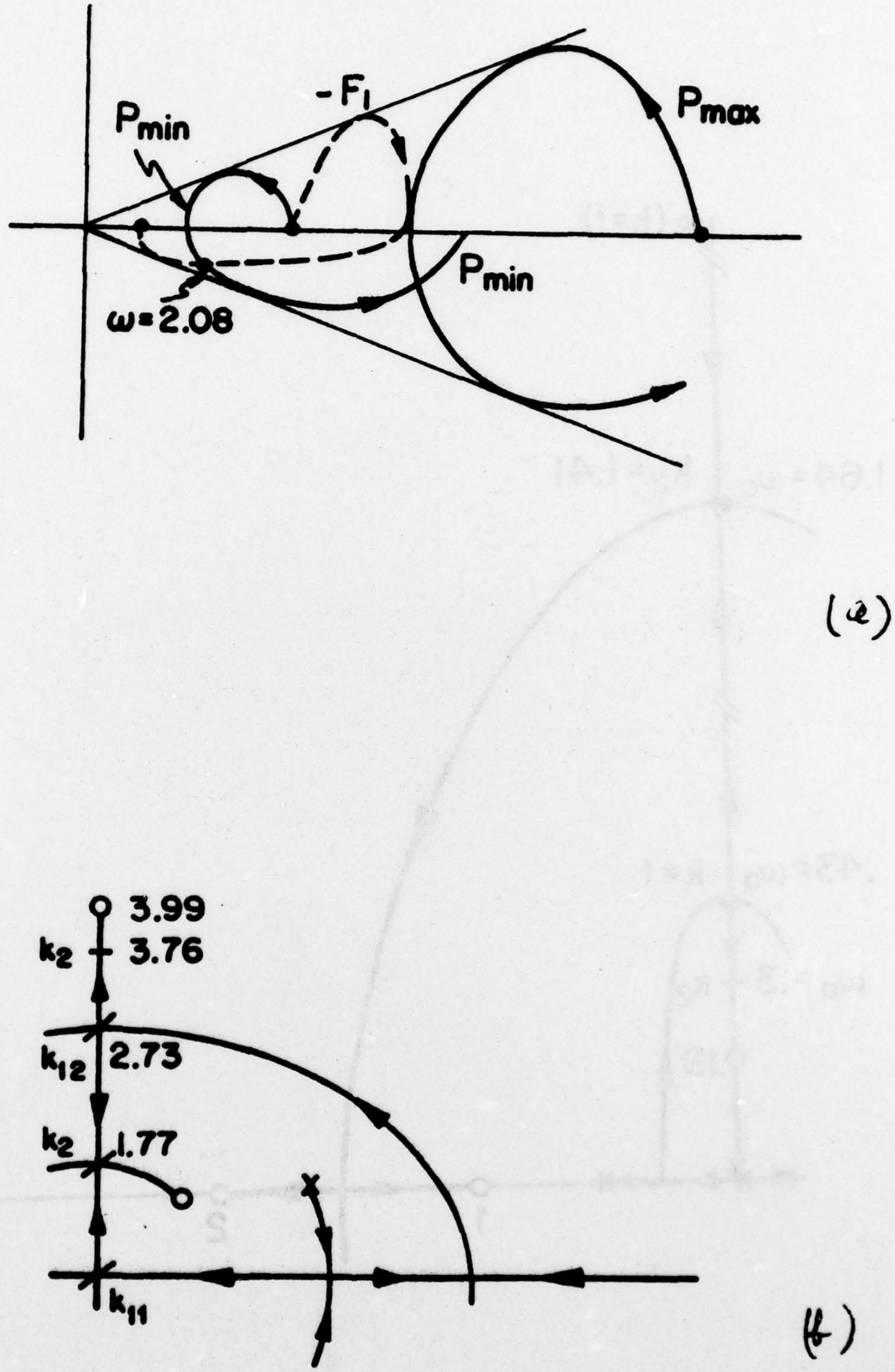


7. (a) Case 3. Nyquist loci of kP_1 and compensations F_1, F_2.

(b) Root loci of $1 + \frac{kP_1}{F_2}$

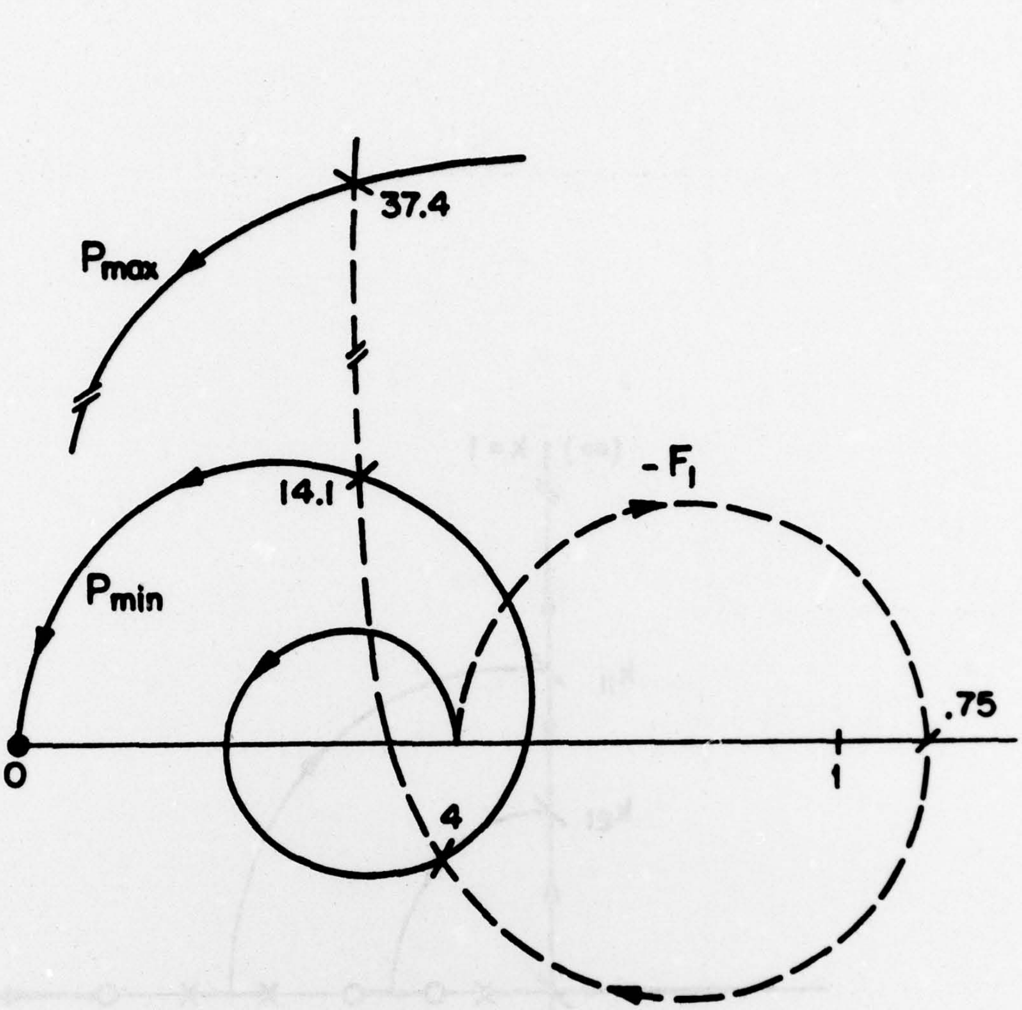


(7b)



8. (a) Nyquist loci of kP_1 and $-F_1$

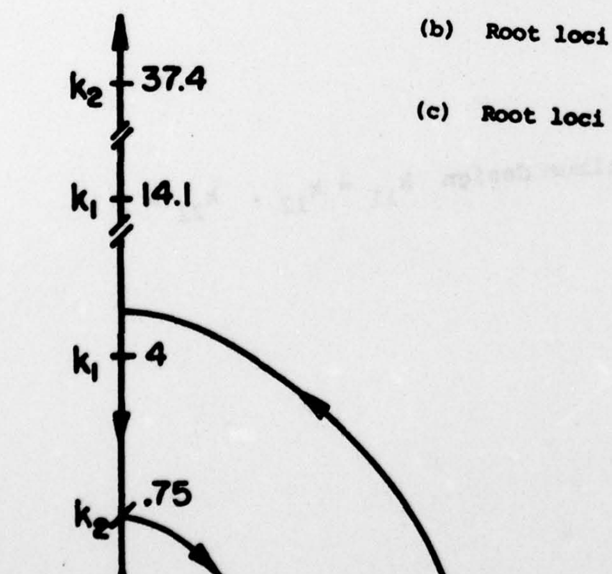
(b) Root locus of $1 + \frac{kP_1}{F_2}$

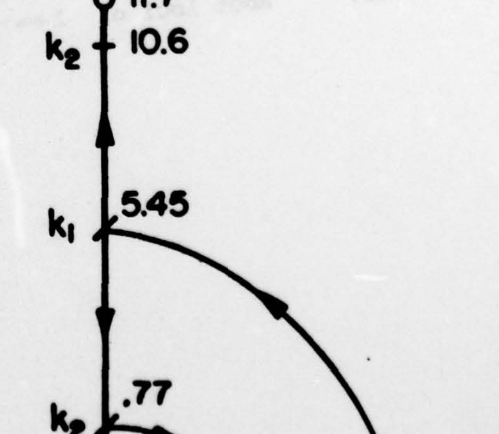


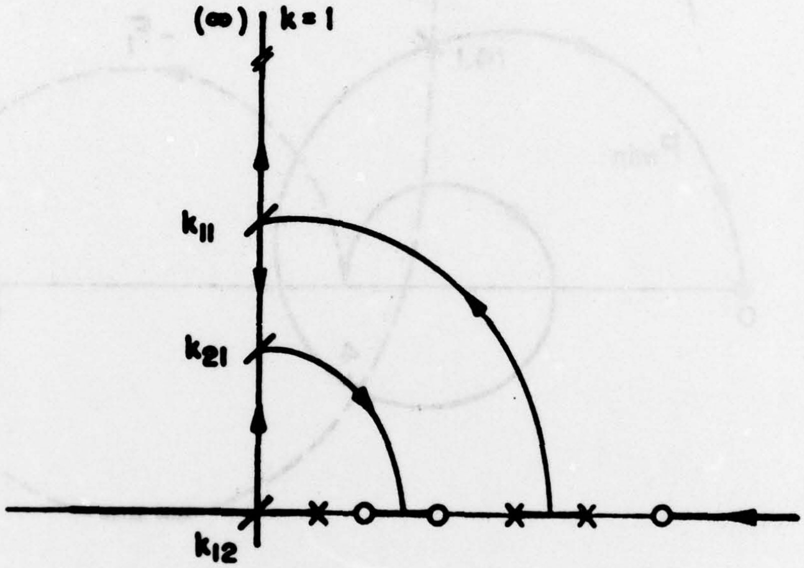
9. (a) Case 4. Nyquist loci of P and $-F_1$.

(b) Root loci of $1 + \frac{kP_1}{F_1}$.

(c) Root loci of $1 + \frac{kP}{F_3}$.







10. Root loci of $1 - \frac{kP_1^*}{F}$. In optimum design $k_{11} = k_{12}$, $k_{21} = 1$.

FIGURE TITLES

1. Plant for which nmp alleviation is possible.

2. Case 1. Nyquist locus of $P = \frac{k(s-z)}{(s-\lambda z)}$, $k \in [1, k_2]$,

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   (b) $k_2 > \lambda > 1$, dynamic F needed.

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5. Reduction of nmp property $\forall k$, by guaranteeing zeros of ψ inside semicircle $W_1, W_2 \cdots W_5$. Root loci of $1 + \frac{kP_1}{F_{opt}}$.

6. Case 2. Nyquist loci of kP_1 and compensations F .

   (a) in complex plane, (b) in Nichols chart,

   (c) Root loci of corresponding $1 + \frac{kP_1}{F_i}$.

7. (a) Case 3. Nyquist loci of kP_1 and compensations F_1, F_2 .

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8. (a) Nyquist loci of kP_1 and $-F_1$

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9. (a) Case 4. Nyquist loci of P and $-F_1$.

   (b) Root loci of $1 + \frac{kP_1}{F_1}$.

   (c) Root loci of $1 + \frac{kP}{F_3}$.

10. Root loci of $1 - \frac{kP_1^*}{F}$. In optimum design $k_{11} = k_{12}$, $k_{21} = 1$.

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The benefits of feedback achievable in nonminimum-phase (nmp) systems are severely limited. However, if there are two parallel nmp branches whose outputs can be measured, then it may be possible to eliminate or reducethe nmp property with respect to some parts of the system, even if each of the two branches has uncertain parameters. It is always possible to do so for a finite, nonzero range of uncertainties for any complexity of pole-zero patterns of the branches. A design philosophy and → next page

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20. Abstract continued

methodology is presented for the general problem, permitting one to approach an optimum solution, under various constraints.